Math 271
Applications of Differentiation

Find all asymptotes without a calculator:

1. \( f(x) = \frac{x - 4}{x - 5} \)

2. \( f(x) = \frac{3x}{x^2 - 4} \)

Find intercepts, asymptotes, relative extrema, and concavity. Use to sketch the following without a calculator:

3. \( f(x) = \frac{x^2}{x - 1} \)

Solve:

4. Find the absolute extrema of \( f(x) = \frac{x^3 - 4x^2 + 7x}{x} \) on the interval \([0, 3]\).

5. Determine whether the Rolle's Theorem can be applied to \( f(x) = x^2 + 3x \) on the interval \([0, 2]\). If it can be applied, find \( c \). If it cannot be applied, tell why.

6. Determine if the Mean Value Theorem can be applied to \( f(x) = \frac{-1}{x} \) on the interval \([-3, -\frac{1}{2}]\). If I can be applied, find \( c \). If it cannot be applied, tell why.

7. Find the relative extrema and where the function is increasing and decreasing if the derivative of a function is \( f'(x) = 4x^3 - 12x^2 + 8x \).

8. Find any points of inflection and discuss concavity for the function \( f(x) = x^4 - 6x^3 \).

9. Find \( dy \) if \( y = \frac{x^2}{2x^2 + 1} \), \( x = 2 \), and \( \Delta x = 0.001 \).

10. Sketch a graph having \( f(0) = 0; \ f'(x) \geq 0 \) for all \( x; \ f''(x) < 0 \) for \( x < 1; \ f''(x) > 0 \) for \( x > 1; \ f''(1) = 0 \).

11. Sketch a graph having a vertical asymptote at \( x = 2; \) a horizontal asymptote at \( y = 0; \ f''(x) > 0 \) for \( x < 2 \) and \( x > 2 \); no intercepts or extrema.

12. \( \lim_{{x \to \infty}} \frac{2x - 7}{x^2 - 1} \)
13. \( \lim_{x \to \infty} \frac{\sin x}{3x} \)

14. \( \lim_{x \to \infty} \frac{6x}{\sqrt{x^2 - 3}} \)

15. An open box is to be made from a rectangular piece of material by cutting equal squares from each corner and turning up the sides. Find the dimensions of the box of maximum volume if the material has dimensions 6in by 6 in.

16. A farmer has 160ft of fencing to enclose two adjacent rectangular pigpens. What dimensions should be used for each pigpen so that the enclosed area will be a maximum?

17. The measurement of the edge of a piece of square floor tile is found to be 12in with a possible error of 0.02in. Use differentials to approximate the maximum possible error in the area of the tile. What is the relative error and the percentage error?